Centreville High School

Geometry Honors Summer Assignment

Name: ____________________________

**Welcome to Geometry!**

**This summer packet is for all students enrolled in Geometry Honors at Centreville High School for Fall 2018. The packet contains prerequisite skills that you will need to be successful in geometry … yes, there's a lot of algebra involved! ☺**

**We will cover each of these skills briefly in class, but it will greatly help you to spend some time this summer keeping the skills and concepts fresh in your mind. Please complete the entire packet on separate notebook paper and bring the packet AND your work with you to class on the FIRST day of school in August. We will review it as a homework assignment and you will be evaluated on the content of this packet during the first 2 weeks of class. Have a great summer and we looking forward to seeing you in August!**

**As you work through the packet, keep track of the following:**

| **"Things I learned, but forgot how to do:"** | **"Things I never learned:"** |
| --- | --- |
| | |

## A. Determining Whether a Point is on a Line

**Example:** Decide whether (3, -2) is a solution of the equation $y = 2x - 8$.

| | |
|---|---|
| -2 = 2(3) – 9 | Substitute 3 for *x* and -2 for *y*. |
| -2 = -2 | Simplify |

The statement is true, so (3, -2) is a solution of the equation $y = 2x - 8$.

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**Exercises: Determine whether each coordinate point is a solution for the given equation.**

1. $y = -10x - 2;\ (1, -12)$
2. $y = \frac{3}{2}x + 10;\ (4, 12)$
3. $9x - y = -4;\ (-1, -5)$
4. $y + 5 = \frac{5}{3}x;\ (9, 10)$

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## B. Calculating Slope

**Example:** Find the slope of a line passing through (3, -9) and (2, -1).

| | |
|---|---|
| $m = \frac{y_2 - y_1}{x_2 - x_1}$ | Formula for slope |
| $m = \frac{-1-(-9)}{2-3} = \frac{-1+9}{-1}$ | Substitute values and simplify |
| $m = \frac{8}{-1} = -8$ | Slope is -8 |

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**Exercises: Calculate the slope of the line passing through each set of coordinate points.**

5. (5, 6) (9, 8)
6. (-6, -4) (1, 10)
7. (14, -5) (7, 8)
8. (-9, 13) (2, -10)

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## C. Writing the Equation of a Line

**Example:** Write an equation of the line that passes through the point (3, 4) and has a y-intercept of 5.

| | |
|---|---|
| $y = mx + b$ | Write the slope-intercept form |
| 4 = 3*m* + 5 | Substitute values for b, x and y; then simplify |
| -1 = 3*m* | |
| $-\frac{1}{3} = m$ | Slope is $m = -\frac{1}{3}$. The equation of the line is $y = -\frac{1}{3}x + 5$ |

**Exercises: Write the equation of the line passing through the following point and y intercept.**

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9. (-3, 10); b = 8

10. (-1, 4): b = -8

11. (5, -8); b = 7

12. (2, 3); b = 2

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## D. Writing the Equation of a Line

**Example:** Write the equation of the line that passes through the points (4, 8) and (3, 1).

$m = \frac{1-8}{3-4}$ Substitute values to find the slope of the line

$m = \frac{-7}{-1} = 7$ Simplify.

1=7(3) + *b* Substitute values into *y* = *mx* + *b* and solve for *b*.

1 = 21 + *b*

-20 = *b* **The equation of the line is y = 7x - 20**

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**Exercises: Write the equation of the line passing through each set of coordinate points.**

13. (5, -1) (4, -5)

14. (1, 2) (-1, -4)

15. (-3, 2) (-5, -2)

16. (-6, 4) (6, -1)

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## E. Distance Formula

**Example: Find the distance between the points (-**4, 3) and (-7, 8).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Substitute coordinate values to find the distance

$= \sqrt{(-7-(-4))^2 + (8-3)^2}$ Simplify.

$= \sqrt{(-3)^2 + (5)^2}$

$= \sqrt{34}$

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**Exercises: Find the distance between the following points:**

17. (-3, 4) (1, 4)

18. (-8, 5) (-1, 1)

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## F. Combining Like Terms

**Example: Simplify.**

$8x^2 + 16xy - 3x^2 + 3xy - 3x$

$8x^2 - 3x^2 + 16xy - 3xy - 3x$ **Group like terms and simplify.**

$5x^2 - 3x + 19xy$

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**Exercises: Simplify.**

19. $-5m + 3q + 4m - q$
20. $-3p - 4t - 5t - 2p$
21. $3x^2y - 5xy^2 + 6x^2y$
22. $5x^2 + 2xy - 7x^2 + xy$

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## G. Solving Equations With Variables on Both Sides

**Example: Solve.**

$6a - 12 = 5a + 9$

$a - 12 = 9$ **Subtract *5a* from each side. Add 12 to each side.**

$a = 21$

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**Exercises: Simplify.**

23. $8m + 1 = 7m - 9$
24. $3a - 12 = -6a - 12$
25. $-7x + 7 = 2x - 11$

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## H. Solving Inequalities

**Example: Solve.** Remember **w**hen you multiply or divide each side of an inequality by a negative number, you must reverse the inequality symbol to maintain a true statement.

a. $5x - 4 \geq 4x + 6$

$x - 4 \geq 6$

$x \geq 10$

b. $10 - 7x < 24$

$-7x < 14$

$x > -2$

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**Exercises: Solve.**

26. $-5 + m < 21$
27. $-3x + 4 \leq -5$
28. $2b + 4 > -3b + 7$
29. $14 - 5t \geq 28$

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## I. Solving Proportions

**Example: Use cross products to solve.**

a. $\frac{x}{8}=\frac{3}{4}$

$4x = 8\cdot 3$

$4x = 24$

$x = 6$

b. $\frac{6}{x+4}=\frac{1}{9}$

$6\cdot 9 = x+4$

$54 = x+4$

$50 = x$

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**Exercises: Use cross products to solve.**

30. $\frac{t}{27}=\frac{4}{9}$
31. $\frac{27}{5}=\frac{3}{x}$
32. $\frac{19}{x}=\frac{9}{5}$
33. $\frac{1}{18}=\frac{5}{-4(x-1)}$
34. $\frac{3}{m+4}=\frac{9}{14}$
35. $\frac{r}{3r+1}=\frac{2}{3}$

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## J. Simplifying Radicals

**Example: Simplify the expression $\sqrt{20}$**

$\sqrt{20}=\sqrt{4}\cdot\sqrt{5}$

$=2\sqrt{5}$

Use Product Property to simplify.

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**Exercises: Solve:**

36. $\sqrt{52}$
37. $\sqrt{40}$
38. $\sqrt{243}$
39. $\sqrt{320}$

## K. Simplifying Radical Expressions

**Example: Simplify the radical expression.**

**a.**

$5\sqrt{3}-\sqrt{3}-\sqrt{2}$

$=4\sqrt{3}-\sqrt{2}$

**b.**

$(2\sqrt{2})(5\sqrt{3})$

$=2\cdot 5\cdot\sqrt{2}\cdot\sqrt{3}$

$=10\sqrt{6}$

**c.**

$\left(5\sqrt{7}\right)^2$

$=5^2\sqrt{7^2}$

$=25\cdot 7$

$=175$

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**Exercises: Solve.**

40. $\sqrt{64}-\sqrt{28}$
41. $\sqrt{242}+\sqrt{200}$
42. $\sqrt{20}+\sqrt{45}-\sqrt{5}$
43. $\sqrt{363}\cdot\sqrt{300}$
44. $\sqrt{21}\cdot\sqrt{24}$
45. $(5\sqrt{4})(2\sqrt{4})$
46. $\left(8\sqrt{3}\right)^2$
47. $\left(10\sqrt{11}\right)^2$

## L. Simplifying Quotients with Radicals

**Example: Simplify the quotient** $\frac{6}{\sqrt{5}}$

$$\frac{6}{\sqrt{5}} = \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

Multiply the numerator and denominator by $\sqrt{5}$ to eliminate

$$= \frac{6\sqrt{5}}{\sqrt{5}\sqrt{5}}$$

the radical in the denominator

$$= \frac{6\sqrt{5}}{5}$$

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**Exercises: Solve:**

48. $\frac{16}{\sqrt{24}}$

49. $\frac{9}{\sqrt{52}}$

50. $\frac{\sqrt{27}}{\sqrt{45}}$

51. $\frac{\sqrt{50}}{\sqrt{75}}$

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**Helpful hints with radicals:** a radical is in simplest form if there are 1) no fractions in the radicand, 2) no perfect squares in the radicand and 3) no radicals in the denominator. LOOK for ways to simplify the fraction BEFORE you rationalize the denominator!

## M. Properties of Exponents

**Example:** An expression like $5^3$ is called a **power.** The **exponent** $^3$ represents the number of times the **base** (5) is used as a factor: $5^3 = 5 \cdot 5 \cdot 5$ (3 factors of 5). To simplify expressions involving exponents, you must use the following properties of exponents:

**CONCEPT SUMMARY — MULTIPLICATION PROPERTIES OF EXPONENTS**

Let $a$ and $b$ be numbers and let $m$ and $n$ be positive integers.

**PRODUCT OF POWERS PROPERTY**

To multiply powers having the same base, add the exponents.

$a^m \cdot a^n = a^{m+n}$ Example: $3^2 \cdot 3^7 = 3^{2+7} = 3^9$

**POWER OF A POWER PROPERTY**

To find a power of a power, multiply the exponents.

$(a^m)^n = a^{m \cdot n}$ Example: $(5^2)^4 = 5^{2 \cdot 4} = 5^8$

**POWER OF A PRODUCT PROPERTY**

To find a power of a product, find the power of each factor and multiply.

$(a \cdot b)^m = a^m \cdot b^m$ Example: $(2 \cdot 3)^6 = 2^6 \cdot 3^6$

**Examples:**

**a:**

$$\left(-3xy^2\right)^3 \cdot y$$
$$= (-3)^3 \cdot x^3 \cdot (y^2)^3 \cdot y^1$$
$$= -27x^3 \cdot y^{6+1}$$
$$= -27x^3y^7$$

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**b:**

$\frac{1}{r^7} g r^4$

$= \frac{r^4}{r^7}$

$= r^{4-7}$

$= r^{-3}$

$= \frac{1}{r^3}$

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**c:**

$\frac{1}{x^6} g \left(\frac{x}{2}\right)^6$

$= \frac{1}{x^6} g \frac{x^6}{2^6}$

$= \frac{x^6}{64x^6}$

$= \frac{1}{64}$

**DEFINITION OF ZERO AND NEGATIVE EXPONENTS**

Let $a$ be a nonzero number and let $n$ be a positive integer.

- A nonzero number to the zero power is 1: $a^0 = 1$, $a \neq 0$.
- $a^{-n}$ is the reciprocal of $a^n$: $a^{-n} = \frac{1}{a^n}$, $a \neq 0$.

**CONCEPT SUMMARY** **DIVISION PROPERTIES OF EXPONENTS**

Let $a$ and $b$ be numbers and let $m$ and $n$ be integers.

**QUOTIENT OF POWERS PROPERTY**

To divide powers having the same base, subtract exponents.

$\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$ **Example:** $\frac{3^7}{3^5} = 3^{7-5} = 3^2$

**POWER OF A QUOTIENT PROPERTY**

To find a power of a quotient, find the power of the numerator and the power of the denominator and divide.

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$ **Example:** $\left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3}$

**Exercises: Simply:**

**52.** $\left(-\frac{2}{3}\right)^3$

53. $\left(\frac{1}{2}ab\right)^5$

54. $2n^4(3n)^2$

55. $\left(8^{-1}\right)^{-3}$

56. $\left(18a^2\right)^2$

57. $\left(\frac{5}{m}\right)^3$

58. $\left(5x \cdot x^3\right)^4$

59. $\left(\frac{x^4}{x^3}\right)^2$

60. $\left(\frac{2x^0}{8y^{-7}}\right)$

61. $\left(5ab^3\right)^2\left(-7b^2c\right)$

62. $4y^3z\left(\frac{y}{2z}\right)^{-3}$

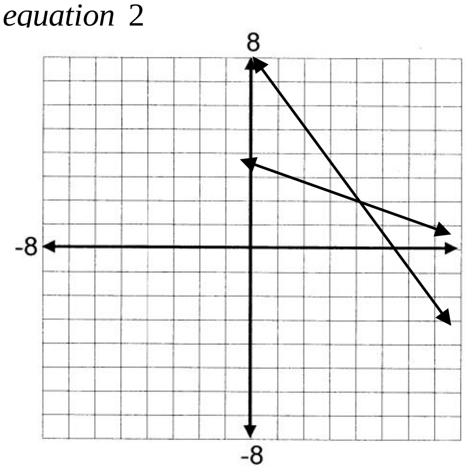
63. $\left(3c^{-4}d^5\right)^{-2}\left(12cd^{-4}\right)$

## N. Solving Systems of Equations

**Example: Use Substitution to solve the linear system:** $3x + 2y = 16$ *equation* 1

$x + 3y = 10$ *equation* 2

**Solve** for $x$ in equation 2 since it is easy to isolate $x$**:** $x = 10 - 3y$
**Substitute** $(10 - 3y)$ for $x$ in Equation 1: $3(10 - 3y) + 2y = 16$.
**Solve** for $y$ to get $y = 2$.
**Substitute** value for $y$ into the equation: $x = 10 - 3(2)$
$x = 4$



The solution is (4, 2), the ordered pair that makes BOTH equations true.

**CHECK:**

- substitute 4 for x and 2 for y in the original equations.
- graph the original equations in the same coordinate plane.
  the graphs should intersect at (4, 2).

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**Exercises: Use Substitution to solve the system of linear equations.**

64. $2x-3y=-16$; $y=5x+1$

65. $x+y=8$; $2x+5y=3$

66. $9x+4y=3$; $x+8y=6$

67. $3x+y=6$; $5(x+y)=22$

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**Example: Use Linear Combinations to solve the linear system:** $4x-3y=-5$; $7x+2y=-16$

**The goal is to obtain coefficients that are opposites for one of the variables.**

| | | | |
|---|---|---|---|
| $4x-3y=-5$ | **multiply by 2** ⟹ | $8x-6y=-10$ | |
| $7x+2y=-16$ | **multiply by 3** ⟹ | $+\ \underline{21x+6y=-48}$ | **Add the equations** |
| | | $29x=-58$ | **Solve for *x*** |
| | | $x=-2$ | |

- **Substitute** -2 for *x*: $4(-2)-3y=-5$.
- **Solve** to get $y=-1$
- The solution is (-2, -1).
- **Check** the ordered pair in the original equations.

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**Exercises: Use Linear Combinations to solve the system of linear equations.**

68. $3x-2y=-6$; $7x-6y=12$

69. $5x+9y=-6$; $2x-6y=6$

70. $8x+2y=13$; $4x+y=11$

71. $x=\frac{1}{2}y+3$; $2x-y=6$

Remember the special case scenarios … parallel lines do not intersect and coincident lines have infinite solutions!

## O. Squaring Binomials

**One way to square a binomial is to remember the patterns:**

**SQUARE OF A BINOMIAL PATTERN**

$(a+b)^2=a^2+2ab+b^2$ **Example:** $(x+4)^2=x^2+8x+16$

$(a-b)^2=a^2-2ab+b^2$ **Example:** $(2x-6)^2=4x^2-24x+36$

**If you have trouble remembering the patterns, you can use the distributive property and FOIL.**

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**Example: Find the product and simplify: (r + 3)$^2$**

**Distributive Property:**

$= (r+3)(r+3)$

$= r(r+3)+3(r+3)$

$= r^2+3t+3r+9$

$= r^2+6r+9$

**FOIL:** (first, outer, inner, last)

$= (r+3)(r+3)$

$= r(r)+(r)3+3(r)+3(3)$

$= r^2+3t+3r+9$

$= r^2+6r+9$

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**Exercises: Find the product:**

72. $(x+2)^2$
73. $(x-1)^2$
74. $(x-0.5)^2$
75. $(15-x)^2$
76. $(10+x)^2$

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## P. Solving ax$^2$ + c = 0

A **quadratic equation** is an equation that can be written in the standard form: $ax^2 + bx + c = 0$ where $a \neq 0$. When b = 0, the quadratic equation has the form $ax^2 + c = 0$. **In this case you can solve for x.**

Solving $ax^2 + c = 0$ for $x^2$ you get $x^2 = \frac{-c}{a}$ and the following rules apply:

- If $\frac{-c}{a} > 0$, then there are two solutions for x: $\pm\sqrt{\frac{-c}{a}}$
- If $\frac{-c}{a} = 0$, then there is one solution for x: 0
- If $\frac{-c}{a} < 0$, then there are no real solutions for x.

**Example: Solve the equation**

**a.** $3x^2-1=23$

$3x^2=24$

$x^2=8$

$x=\pm\sqrt{8}$

$x=\pm 2\sqrt{2}$

**b.** $12-x^2=13$

$-x^2=1$

$x^2=-1$

$\textit{no real solution}$

**c.** $4+2n^2=4$

$2n^2=0$

$n^2=0$

$n=0$

**Exercises: Solve for x:**

77. $x^2=289$
78. $x^2-7=6$
79. $6x^2=294$
80. $9x^2+7=52$

## Q. Factoring and solving a quadratic expression of the form $x^2 + bx + c$

- To **factor** a quadratic expression means to write it as the product of two linear expressions.
- To factor $x^2 + bx + c$, you need to find numbers $p$ and $q$ such that:

$$p + q = b \quad \textit{and} \quad pq = c$$

- Remember, $x^2 + bx + c = (x+p)(x+q)$ when $p + q = b$ and $pq = c$

**Exercises: Factor the Trinomial**

81. $x^2 + 5x + 6$
82. $x^2 + 6x + 5$
83. $x^2 - 5x + 6$
84. $x^2 - 3x + 2$
85. $x^2 - 7x + 12$

**Challenge:**

86. $8x^2 - 2x - 3$
87. $3x^2 + 13x - 14$
88. $5x^2 + 27x - 18$
89. $6x^2 - 7x + 1$

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## R. Solving $ax^2 + bx + c = 0$

You can solve any quadratic equation by using the **quadratic formula.** This formula states that the solutions of the quadratic equation $ax^2 + bx + c = 0$ are: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ when $a \neq 0$ and $b^2 - 4ac \geq 0$

**Example: Solve $x^2 - 4x - 12 = 0$ by using the quadratic formula.**

**Substitute a = 1, b = -4, c = 12 into the quadratic formula.**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2}$$

$$x = \frac{4 \pm \sqrt{64}}{2}$$

**The solutions are: 6 and -2.** (Check by substituting into the equation.)

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**Exercises: Use the quadratic formula to solve each equation.** Round solutions to the nearest $100^{th}$ .

90. $a^2 + 8 = 6a$
91. $-25 = x^2 + 10x - 5$
92. $4x^2 - 3x = 7$

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