AP Calculus AB

SUMMER ASSIGNMENT - Part 1

Dear future Calculus AB student –

We are excited to work with you next year in Calculus AB☺. In order to help you be prepared for this class, please complete the summer assignment. We would like to recommend that you do this Short Answer Section first. In that packet there are directions and review of the main topics that you need for this packet as well as for the multiple choice packet. Do all work on these packets. **Show work to support your answers.** These problems are <u>due the first day of class</u>. On the first day of class you will have a short amount of time to ask questions before turning these packets in. Do not expect there to be time in class for ALL questions to be resolved. **You are responsible for understanding all of the material in this packet.**

If you have any questions, please do not hesitate to email either of us.

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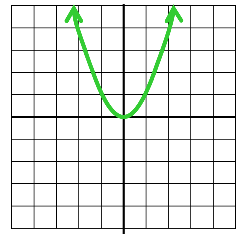
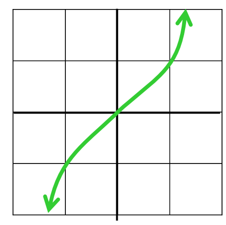
See you in August!
Mr. Mossholder and Mrs. Rigby

SHORT ANSWER SECTION

<u>Objectives:</u> For objectives #1 - #5 you should be able to do <u>all without a calculator</u>

1. Identify functions as even or odd
   - Algebraically
   - Graphically
2. Know key points and basic shapes of essential graphs
   - $f(x)=\sqrt{x}$, $f(x)=x^2$, $f(x)=x^3$
   - $f(x)=e^x$
   - $f(x)=\ln(x)$
   - $f(x)=\sin x$, $f(x)=\cos x$
3. Review Trigonometry
   - Evaluate trig values
   - Evaluate inverse trig values
   - Solve trig equations
4. Understand characteristics of rational expressions (be able to sketch **without** a calculator)
   - Vertical asymptotes
   - Horizontal asymptotes
   - Zeros
   - Holes
5. Be able to use properties of natural logarithms to solve equations
6. Know your calculator

I. Symmetry – Even and Odd Functions

Quick Review		
Even Function	Symmetric about the y axis $f(-x) = f(x)$ for all x	Example: $y = x^2$ 
Odd Function	Symmetric about the origin (equivalent to a rotation of 180 degrees) $f(-x) = -f(x)$ for all x	Example: $y = x^3$ 

To determine algebraically if a function is even, odd, or neither, find $f(-x)$ and determine if it is equal to $f(x)$, $-f(x)$, or neither.

Example: Determine if $f(x) = \frac{4x}{x^2+1}$ is even or odd.

$f(-x) = \frac{4(-x)}{(-x)^2+1} = \frac{-4x}{x^2+1} = -\frac{4x}{x^2+1} = -f(x)$ Therefore, $f(x)$ is an odd function.

Determine if the following functions are even, odd, or neither.

1. $f(x) = \frac{x^2}{x^4+3}$

2. $f(x) = \frac{x}{x+1}$

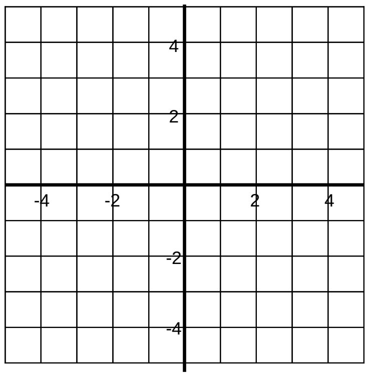
3. $f(x) = 1 + 3x^2 + 3x^4$

4. $f(x) = 1 + 3x^3 + 3x^5$

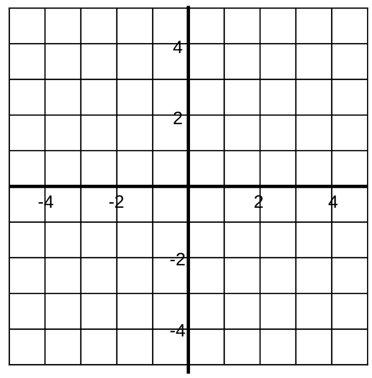
II. Essential Graphs

For each graph, show two key points (label coordinates) and basic shape of the graph.

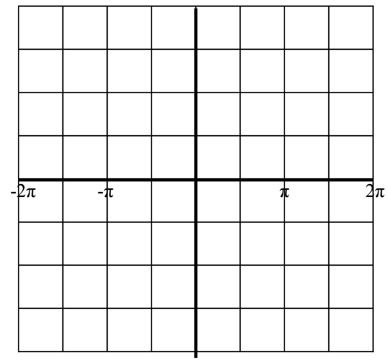
1. $f(x) = \sqrt{x}$



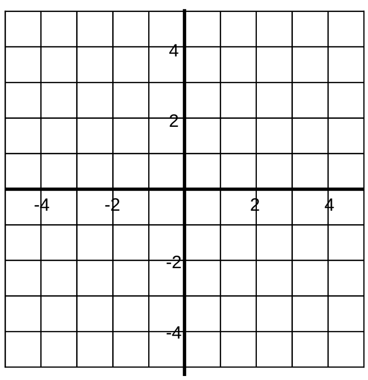
2. $f(x) = x^3$



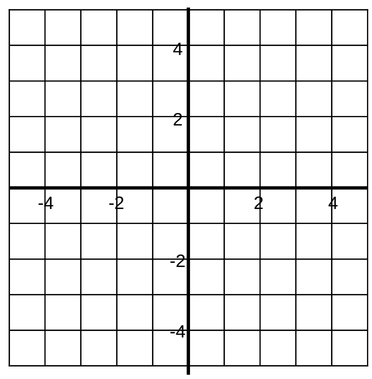
3. $f(x) = \sin x$



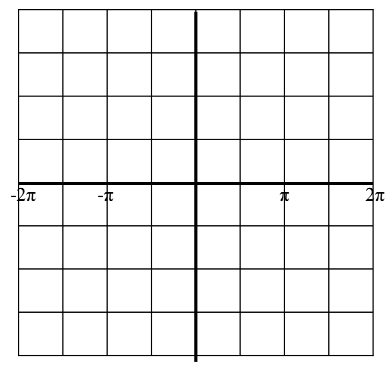
4. $f(x) = x^2$



5. $f(x) = \frac{1}{x}$

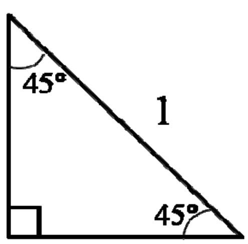


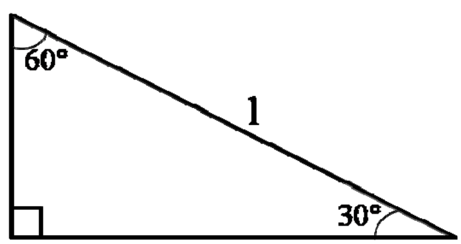
6. $f(x) = \cos x$



III. Trigonometry Review

On each right triangle with a hypotenuse of 1, label the lengths of the other sides:





45° = _____ radians 90° = _____ radians 30° = _____ radians 60° = _____ radians

⇨ In calculus we always use radians! Never degrees.

Make sure you know the following identities:

Pythagorean identities: $\sin^2 x + \cos^2 x = 1$ $1 + \tan^2 x = \sec^2 x$ $\cot^2 x + 1 = \csc^2 x$

Double angle identities: $\sin 2x = 2\sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$

1. Evaluate the sine, cosine, and tangent of each angle without using a calculator. It is not necessary (ever again!) to rationalize the denominator. (ie, an answer of $\frac{1}{\sqrt{2}}$ does not need to be written $\frac{\sqrt{2}}{2}$)

a) $\frac{\pi}{4}$

b) $\frac{5\pi}{4}$

c) $-\frac{\pi}{6}$

d) $\frac{\pi}{2}$

e) $\frac{5\pi}{3}$

f) $\frac{11\pi}{6}$

2. Find two solutions to the inverse trig equations. $(0 \le \theta \le 2\pi)$.
   1. Draw a triangle, if necessary
   2. Label θ and sides
   3. Determine reference angle
   4. Notice restrictions
   5. Place in correct quadrant

a) $\cos\theta = \frac{\sqrt{2}}{2}$

b) $\cos\theta = -\frac{\sqrt{2}}{2}$

c) $\sec\theta = 2$

d) $\tan\theta = 1$

e) $\cot\theta = -\sqrt{3}$

f) $\sin\theta = -\frac{\sqrt{3}}{2}$

3. Solve for y. Notice the only difference between question #2 and #3 is the domain restrictions #3. Inverse trig functions have one answer due to domain restrictions. Note: $\sin^{-1}x$ is the same as arcsinx

a) $y = \sin^{-1}\left(\frac{1}{2}\right)$

b) $y = \cos^{-1}\left(\frac{1}{2}\right)$

c) $y = \arctan\left(\sqrt{3}\right)$

d) $y = \csc^{-1}(2)$

e) $y = \arccos(0)$

f) $y = \cot^{-1}(-1)$

IV. Rational Functions

Rational functions are ratios of polynomials: $h(x)=\frac{f(x)}{g(x)}$

$h(x)$ has a **zero** when $h(x)=0$ (which occurs when $f(x)=0$ and the factor does not cancel)

Ex. $h(x)=\frac{x^2+x-2}{x^2-1}$ $\quad$ $h(x)=\frac{(x+2)(x-1)}{(x+1)(x-1)}$ $\quad$ $h(x)=\frac{(x+2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

Therefore, $h(x)=0$ when $x=-2$.

$h(x)$ has a **vertical asymptote** when $g(x)=0$ and the factor that causes $g(x)=0$ does not cancel

Ex. $h(x)=\frac{x^2+x-2}{x^2-1}$ $\quad$ $h(x)=\frac{(x+2)(x-1)}{(x+1)(x-1)}$ $\quad$ $h(x)=\frac{(x+2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

Therefore, $h(x)$ has a vertical asymptote when $x=-1$.

$h(x)$ has a **hole** (is undefined but the limit exists) when $g(x)=0$ and the factor that causes $g(x)=0$ cancels from both $f(x)$ and $g(x)$.

Ex. $h(x)=\frac{x^2+x-2}{x^2-1}$ $\quad$ $h(x)=\frac{(x+2)(x-1)}{(x+1)(x-1)}$ $\quad$ $h(x)=\frac{(x+2)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

Therefore, $h(x)$ has a hole when $x=1$.

Note that $h(x)\neq\frac{(x+2)}{(x+1)}$ because these two functions do not have the same domain.

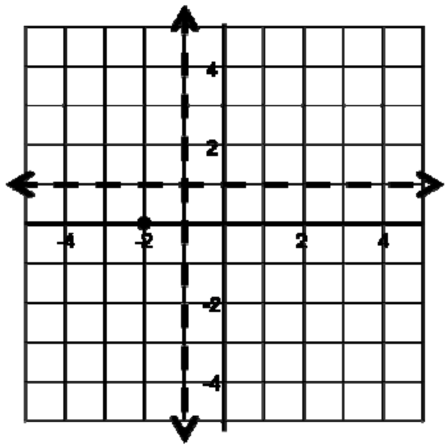
$h(x)$ has a **horizontal asymptote** at $y=a$ when $\lim_{x\to\infty}h(x)=a$ or $\lim_{x\to-\infty}h(x)=a$. To determine $\lim_{x\to\infty}h(x)$ consider first the largest exponent of $f(x)$ and $g(x)$. If $f(x)$ has the larger exponent, then $\lim_{x\to\infty}h(x)=\infty$ (DNE). If $g(x)$ has the larger exponent, then $\lim_{x\to\infty}h(x)=0$. If the exponents are the same, consider the leading coefficient.

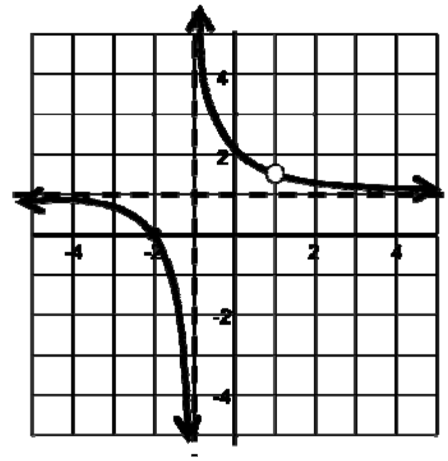
Ex. $h(x)=\frac{x^2+x-2}{x^2-1}$ $\quad$ Leading coefficients $=\frac{1}{1}$

Therefore, $\lim_{x\to\infty}h(x)=1$ and $h(x)$ has a horizontal asymptote at $y=1$.

Once the basic characteristics of rational expressions are determined, the functions can be sketched without a calculator:

Ex. $h(x) = \dfrac{x^2 + x - 2}{x^2 - 1}$





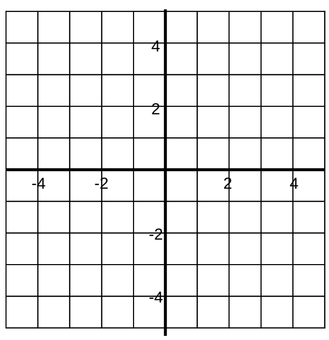
Zero at $x = -2$.
Vertical Asymptote: $x = -1$
Hole when $x = 1$
Horizontal Asymptote: $y = 1$

Graph points as needed until you see the shape.

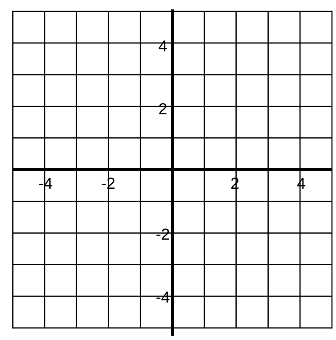
For each rational function below:

a) Find all zeros
b) Write the equations of all vertical asymptotes
c) Write the equations of all horizontal asymptotes
d) Find the x value of any holes
e) Sketch the graph (no calculator) showing all characteristics listed
(Not all functions will have all characteristics listed above)

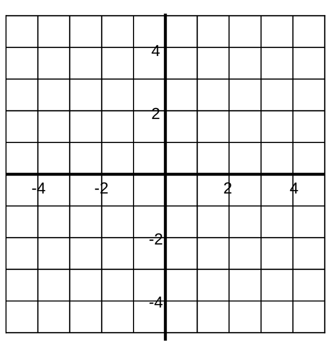
1. $f(x) = \dfrac{x+2}{x-1}$



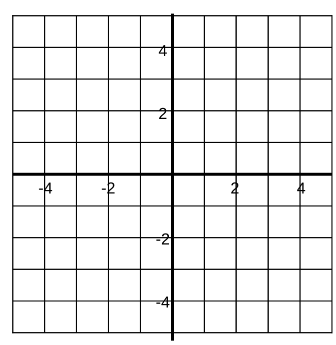
2. $f(x) = \dfrac{2x-3}{x+1}$



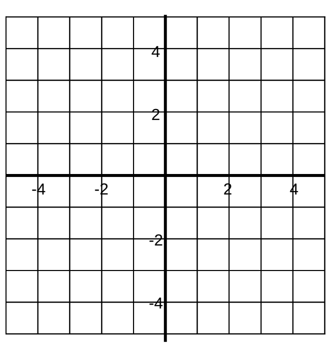
3. $f(x) = \dfrac{1}{x^2}$



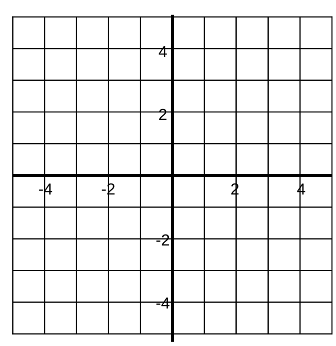
4. $f(x) = \dfrac{x-1}{x^2}$



5. $f(x) = \dfrac{x^2 + 3x - 4}{x^2 - x - 6}$



6. $f(x) = \dfrac{x}{x^2 - 4}$



V. Properties of Natural Logarithms

Recall that $y=\ln(x)$ and $y=e^x$ (exponential function) are inverse to each other

Properties of the Natural Log:

$\ln(AB)=\ln(A)+\ln(B)$ Ex: $\ln(2)+\ln(5)=\ln(10)$

$\ln\left(\dfrac{A}{B}\right)=\ln(A)-\ln(B)$ Ex: $\ln(6)-\ln(2)=\ln\left(\dfrac{6}{2}\right)=\ln(3)$

$\ln(A^p)=p\ln(A)$ Ex: $\ln(x^4)=4\ln(x)$ and $3\ln(2)=\ln(2^3)=\ln(8)$

Know these without a calculator: $\ln(e^x)=x$, $\ln(e)=1$, $\ln(1)=0$, $e^0=1$

Use the properties of natural logs to solve for x.

Ex:

$$5^x=7e^x$$
$$\frac{5^x}{7}=\ln e^x$$
$$\ln\left(\frac{5^x}{7}\right)=\ln e^x$$
$$\ln(5^x)-\ln(7)=x$$
$$x\ln(5)-\ln(7)=x$$
$$x\ln(5)-x=\ln(7)$$
$$x(\ln(5)-1)=\ln(7)$$
$$x=\frac{7}{\ln(5)-1}$$

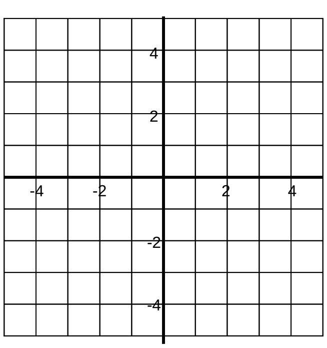
1. $10=4^x$

2. $2e^{3x}=4e^{5x}$

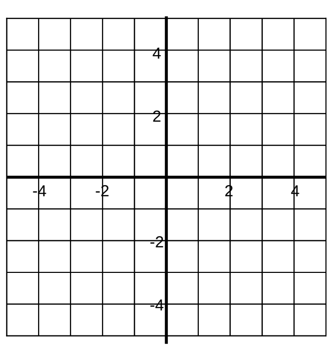
3. $10^{x+3}=5e^{7-x}$

Draw a sketch of each function. Make sure to label the key point and an asymptote.

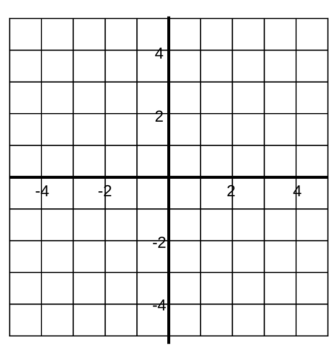
4. $y=2^x$



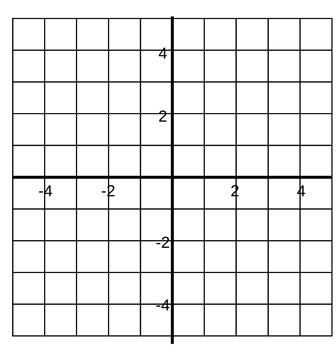
5. $y=e^x$



6. $y=-e^x$



7. $y=\ln x$



VI Calculator Skills

No matter what type of calculator you own, you should be able to do the following problems. Use the index of the owner's manual of your calculator to look up instructions or ask a classmate how to do each.

<u>Evaluations:</u>

1) Fractions and more. Be careful to use enough parentheses.

a) $\dfrac{6-.379}{3+\sqrt{2}}$ b) $\sqrt{2+\pi}$

2) Logarithms and Exponents

a) ln 2 b) log 2 c) ln (3e) d) 5^{-4} e) e^{3}

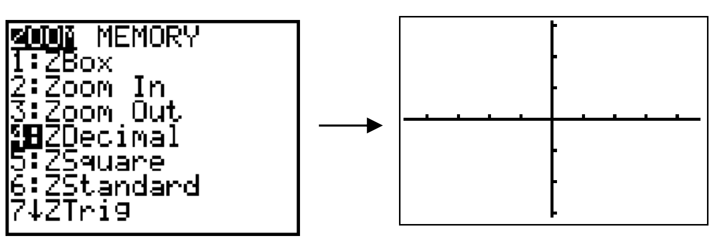
f) $\sqrt[3]{10}$ f) -18^{2} g) $(-18)^{2}$ h) $3^{(2)(.0134)}$

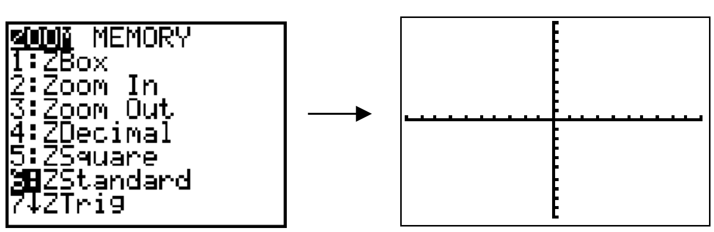
3) Trigonometry Evaluations. Make sure you watch the mode – degree versus radians.

a) $\sin 23^{\circ}$ b) $\sin\dfrac{\pi}{5}$ c) $\sec\dfrac{\pi}{7}$ d) $\tan^{-1}(1.5)$ in radians & degrees

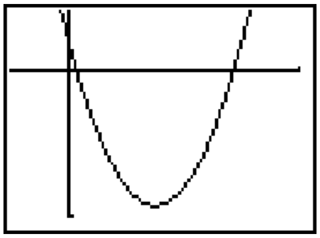
<u>Graphing Skill #1:</u> You should be able to graph a function in a viewing window that shows the important features. You should be familiar with the built-in zoom options for setting the window such as zoom-decimal and zoom-standard. You should also be able to set the window conditions to values you choose.

1. Graph $y = x^{2} - 3$ using the built in zoom-decimal and zoom-standard options. Draw each.





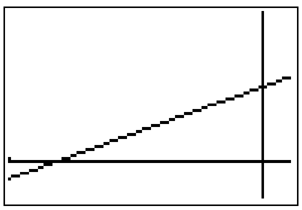
2. Find the appropriate viewing window to see the intercepts and the vertex defined by $y = x^2 - 11x + 10$. Use the window editor to enter the x and y values.



Window: Xmin = ________
Xmax = ________
Xscl = ________
Ymin = ________
Ymax = ________
Yscl = ________

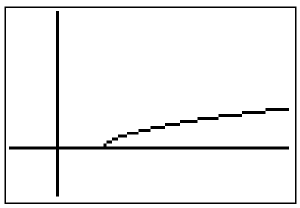
3. Find the appropriate viewing windows for the following functions:

$y = 10 + \frac{1}{4}x$



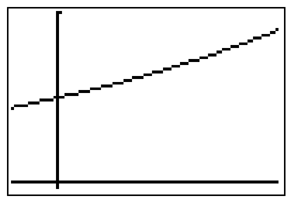
Xmin = ________
Xmax = ________
Xscl = ________
Ymin = ________
Ymax = ________
Yscl = ________

$y = \sqrt{x - 5}$



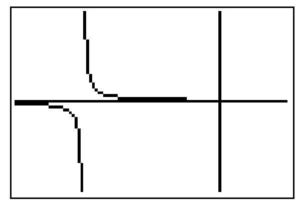
Xmin = ________
Xmax = ________
Xscl = ________
Ymin = ________
Ymax = ________
Yscl = ________

$y = 100(1.06)^x$



Xmin = ________
Xmax = ________
Xscl = ________
Ymin = ________
Ymax = ________
Yscl = ________

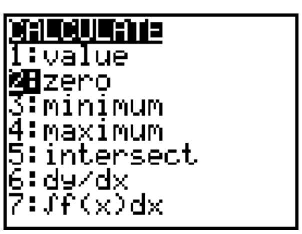
$y = \frac{1}{x + 10}$

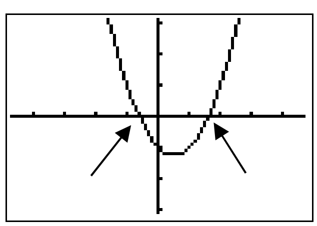


Xmin = ________
Xmax = ________
Xscl = ________
Ymin = ________
Ymax = ________
Yscl = ________

<u>Graphing Skill #2:</u> You should be able to graph a function in a viewing window that shows the x-intercepts (also called roots and zeros). You should be able to accurately estimate the x-intercepts to **3 decimal places.** Use the built-in root or zero command.

1**.** Find the x-intercepts of $y = x^2 - x - 1$. Window [-4.7, 4.7] x [-3.1, 3.1]





(Write intercepts as points)

x-intercepts: _______________

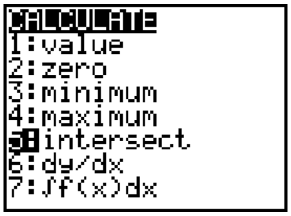
2. Find the x-intercepts of $y = x^3 - 2x - 1$.

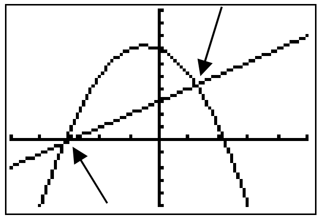
x-intercepts: _______________

<u>Graphing Skill #3:</u> You should be able to graph two functions in a viewing window that shows the intersection points. Sometimes it is impossible to see all points of intersection in the same viewing window. You should be able to accurately estimate the coordinates of the intersection points to **3 decimal places.** Use the built-in intersection command.

1. Find the coordinates of the intersection points for the functions:

$$f(x) = x + 3 \qquad g(x) = -x^2 - x + 7 .$$





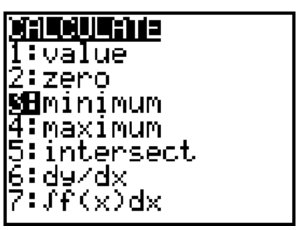
Intersection points: _______________

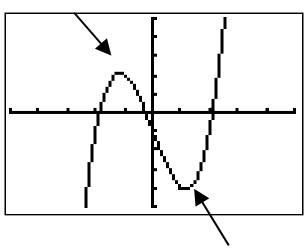
2. Find the coordinates of the intersection points of: $f(x) = 4x^2 \qquad g(x) = 2^x$.

Intersection points: _______________

<u>Graphing Skill #4:</u> You should be able to graph a function and estimate the local maximum or minimum values to **3 decimal places.** Use the built-in max/min command.

1. Find the maximum and minimum values of the function $y = x^3 - 4x - 1$.





(Value means the y-value)

Minimum value: _______________

Maximum value: _______________

2. Find the maximum and minimum values of the function $y = x^3 - 4x^2 + 4x$.

<u>REVIEW:</u> Do the following problems on your calculator.

1. $\dfrac{\sqrt{-4 + 8.556}}{2 + \sqrt[3]{9}}$

2. $\ln 4e^{\sqrt{12}} - 1.32$

3. $\sin 48°$

4) $\cos^{-1}\left(-.75\right)$ in radians and degrees

5) $\tan \dfrac{5\pi}{6}$

6) Find the x-intercepts, relative maximum, and relative minimum of $y = x^3 + 2x^2 - 1$

7) Find the coordinates of the intersection points for the functions $f(x) = 2x^2 + x - 9$ and $g(x) = -\dfrac{3}{4}x + 3$